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Thermal Force Modeling for Global Positioning System Satellites Using the Finite Element Method

Yvonne Vigue*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109
and
Bob E. Schutz† and P. A. M. Abusali‡
University of Texas at Austin, Austin, Texas 78712

Geophysical applications of the Global Positioning System (GPS) require the capability to estimate and propagate satellite orbits with high precision. An accurate model of all the forces acting on a satellite is an essential part of achieving high orbit accuracy. Methods of analyzing the perturbation due to thermal radiation and determining its effects on the long-term orbital behaviour of GPS satellites are presented. The thermal imbalance force, a nongravitational orbit perturbation previously considered negligible, is the focus of this paper. The Earth's shadowing of a satellite in orbit causes periodic changes in the satellite's thermal environment. Simulations show that neglecting thermal imbalance in the satellite force model gives orbit errors larger than 10 m over several days for eclipsing satellites. This orbit mismodeling can limit accuracy in orbit determination and in estimation of baselines used for geophysical applications.

Nomenclature

A = surface area
C_p = specific heat
c = speed of light
E_r = energy emitted by a real body, summed over all wavelengths
f = thermal imbalance force per unit area
h = incident solar radiation received by solar panel
K = thermal conductivity
m = satellite mass, in kg
$\hat{n}$ = unit vector normal to surface of solar panel
q = radiative energy
$\mathbf{r}$ = geocentric satellite position vector
r = radial distance from Earth center-of-mass to satellite
T = temperature
t = time
α = surface absorptivity
ε = emissivity
ρ = density
ψ = solar constant
σ = Stefan-Boltzmann constant
μ = Earth's gravitational constant

Subscripts

a = solar panel sun-tracking front side
b = solar panel back side, no direct sunlight
in = incoming to the spacecraft
out = leaving the body
r = radiative
therm = thermal imbalance

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*Member of Technical Staff. Member AIAA.
†Professor, Associate Director, Center for Space Research.
‡Research Associate, Center for Space Research.

Introduction

MISMODELING of satellite force parameters can have a significant effect on satellite orbits, especially in orbit prediction.[1] Some applications require the capability to estimate and propagate satellite orbits with high precision. TOPEX precision orbit determination is one example where precise modeling of nongravitational forces is essential to fulfill mission requirements.[2] Also, some of the observed drag and orbit decay on the spacecraft LAGEOS has been attributed to unmodeled thermal forces.[3,4] The focus of this analysis was to assess the effects of thermal reradiation and mismodeling of nongravitational forces on satellite orbits. To achieve a high level of orbit accuracy, an accurate model of all the forces acting on an Earth-orbiting satellite is necessary.

Radiative heat transfer between a satellite and its environment is the basis for the thermal force model. A satellite in Earth orbit is continuously illuminated by radiation, most of which comes from the sun. The thermal imbalance force is directly related to the temperature distribution of the satellite in its changing environment. An uneven temperature distribution causes surfaces to reradiate energy at different rates. Some studies have shown that most of the thermal gradient forces on a TOPEX satellite originate within the spacecraft body,[2] whereas other analyses have shown that the dominant source for thermal reradiation forces on a GPS-like satellite is the solar panels, because of their large exposed area and low heat capacity.[5]

The satellite's heated body reradiates energy at a rate that is proportional to its temperature, losing the energy in the form of photons. By conservation of momentum, a net momentum flux out of the body creates a reaction force against the radiating surface, and the net thermal force can be observed as a small perturbation that affects long-term orbital behavior of the spacecraft.[5] The partial differential equations and boundary conditions describing the temperature distribution and the heat transfer between surfaces, along with the application of the finite element method, are presented in this paper. A brief description of the statistical estimation technique used for studying the effect of the thermal imbalance force on satellite orbits is included.

Radiation and Heat Conduction Formulation

Two types of heat transfer that affect a spacecraft in orbit are radiation and heat conduction. The exchange of energy between the spacecraft and its surroundings is described by radiation heat transfer. Conduction is the transfer of heat by molecular motion

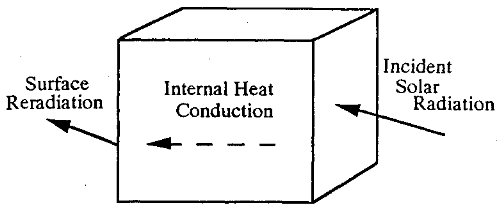

Fig. 1 General heat transfer diagram for a spacecraft.

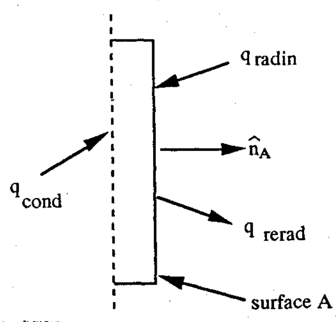

Fig. 2 Conservation of energy diagram for a satellite surface.

within a solid medium. Figure 1 shows the type of heat transfer that affects an orbiting spacecraft.

The rate of radiant energy transfer is given by Stefan-Boltzmann law[6]:

$$E_r = \varepsilon\sigma T^4 \tag{1}$$

By conservation of momentum, the thermal force, or rate of change of momentum for a radiating surface element, assuming a Lambertian surface, is expressed as[5]:

$$\mathrm{d}f_{\text{therm}} = -\frac{2}{3}\frac{\varepsilon\sigma T_a^4}{c}\mathrm{d}A\hat{n}_a \tag{2}$$

The unit vector in Eq. (2) is defined as normal to and directed out of the sun-tracking surface of the solar panel. The differential force must be integrated over the entire surface to determine the complete thermal force:

$$f_{\text{therm}} = -\frac{2\sigma}{3c}\int_\Omega \varepsilon T_a^4 \mathrm{d}A\hat{n}_a \tag{3}$$

Clearly, thermal force cannot be computed unless spacecraft surface temperatures are known.

In general, the temperature at any point within a body satisfies the heat equation[5]:

$$K\,\nabla^2 T = \rho C_p \frac{\partial T}{\partial t} \tag{4}$$

The solution to this second-order partial differential equation requires that boundary conditions be specified. The boundary conditions are defined by thermal radiation and heat conduction. As given by the conservation of energy principle, the total amount of energy coming into a surface is equal to the total amount of energy leaving the surface, assuming there is no internally generated or stored energy (no sinks and no sources) The boundary condition for the satellite surface can be obtained by using this condition as

$$q_{\text{in}} = q_{\text{out}} \tag{5}$$

where q_{in} is the amount of incoming radiative energy due to external sources and internal conduction and q_{out} is the amount of radiative energy leaving the boundary due to reradiation and conduction. Figure 2 shows the conservation of energy principle for a satellite solar panel surface.

Using this concept, the boundary conditions for each surface were constructed. The incident radiative solar energy received per unit area per unit time by side a and side b of the solar panel are represented by h_a and h_b (Ref. 5):

Table 1 GPS thermal and orbit parameters

Model parameter	Value
Initial orbit radius	26,550,000 m
Surface emissivity, ε_a	0.78
Surface emissivity, ε_b	0.83
Surface absorptivity, α_a	0.77–14.1% (panel efficiency)
Solar panel surface area, A	10.832 m^2
Satellite mass, m	845 kg
Initial panel temperature ($t = 0$)	300 Kelvin
Stefan-Boltzmann constant, σ	5.6699 E − 08 W/m^2 K
Speed of light, c	2.998 E + 08 m/s
Solar constant, ψ	1368.2 W/m^2
Total panel thickness (8 layers)	0.01478 m = 0.582 in.

$$KA\frac{\partial T_b}{\partial X} = \varepsilon_b\sigma A T_b^4 - h_b A \tag{6a}$$

$$-KA\frac{\partial T_a}{\partial X} = \varepsilon_a\sigma A T_a^4 - h_a A \tag{6b}$$

The actual amount of incident radiative energy received by each side of the solar panel is a function of panel orientation and the orbit of the satellite. The subscript a represents the left boundary in local coordinates (cold side) and b is the right boundary, which is assumed to be continuously facing the sun during orbit for a Global Positioning System- (GPS-) type satellite. The term on the left side of the equal sign in Eqs. (6a) and (6b) is the heat flux, energy per unit time per unit area, in the local x direction, which is perpendicular to the solar panel face. The values used for some of the parameters just described are shown in Table 1, and are consistent with values used for the GPS satellites.

PDE-Protran, a finite element method program, was used to solve the transient heat conduction and radiation problem presented here. PDE-Protran was developed by Granville Sewell and is a general-purpose two-dimensional partial differential equation solver.[7] This software was combined with a program that incorporated material properties, the satellite's orbit orientation, and thermal environment to determine solar panel surface temperatures. Grid points were chosen to divide the solar panel into small sections or "elements" where the temperature of the solar panel was computed for each grid point (in one dimension, across the thickness of the solar panel.) These grid points coincide with the boundaries between each layer of the solar panel's "sandwiched" materials (shown in Table 2). Accurate and current knowledge of physical parameters such as surface emissivity, thermal conductivity, heat capacity, and material density is required. For this analysis, these material properties are assumed to remain constant throughout the satellite's orbit and only the solar radiation environment varies with time as the satellite experiences eclipsing, or shadowing, from the sun by the Earth. The material parameters directly influence the thermal forces that are calculated and have an effect on the prediction and propagation of the spacecraft trajectory. Also, these material properties may change in time or degrade due to the harsh environment of space.[8]

Orbit Analysis Technique

In this investigation, the equations of motion for an Earth satellite are assumed to include the two-body gravitational effect and the thermal imbalance forces only, and are given in vector form by

$$\ddot{r} = -\frac{\mu r}{r^3} + \frac{f_{\text{therm}}}{m} \tag{7}$$

and thermal imbalance force perturbing the satellite, f_{therm}, is computed as

$$f_{\text{therm}} = \frac{2\sigma A}{3c}(\varepsilon_b T_b^4 - \varepsilon_a T_a^4)\hat{n}_a \tag{8}$$

Table 2 GPS block II solar panel properties

Panel layer composition	Thickness, m	Density, kg/m^3	Specific heat, J/kg K	Conductivity, W/m K
Coverglass	0.00749	2186.622	753.624	1.417
Adhesive	0.00005	1079.472	1256.04	0.116
Solar cell	0.00025	2684.84	711.756	147.994
Interconnect cell adhesive	0.00018	1051.793	1256.04	0.116
Kapton cocured	0.000076	1162.508	1130.436	0.1506
Graphite epoxy	0.00019	2186.622	1373.27	0.8706
Aluminum core	0.00635	24.91088	1046.7	250.966
Graphite epoxy	0.00019	2186.622	1373.27	0.8706

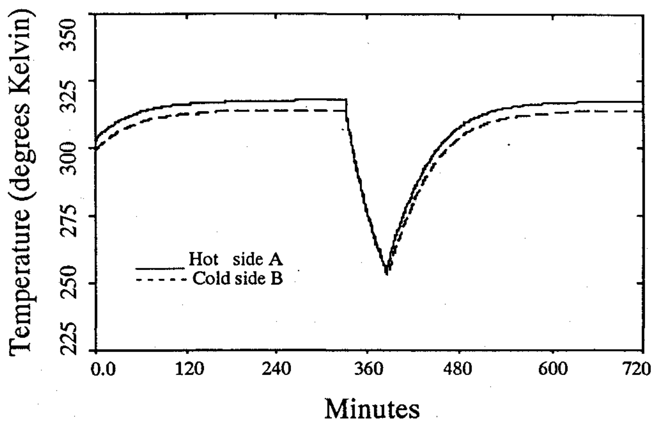

Fig. 3 Temperature history simulation for a GPS solar panel.

Table 3 Coverglass thermal simulation

Test case 1	Test case 2
$K = 1.417$ W/m K	$K = 0.04327$ W/m K
Hot side $T_a = 317.41$ K	Hot side $T_a = 340.30$ K
Cold side $T_b = 313.66$ K	Cold side $T_b = 285.37$ K
Thermal acceleration = 1.88 E − 10 m/s^2	Thermal acceleration = −8.01 E − 9 m/s^2

The effect of the thermal imbalance force on a satellite can be observed by comparing the perturbed orbit with the unperturbed two-body orbit in time. Because there is no closed-form analytical solution for the perturbed equations of motion, a numerical integration technique was necessary to solve the ordinary differential equations of motion. The perturbed and unperturbed orbits originate with the same initial conditions and then the displacement between them at a given time can be observed. A least squares estimation technique is used to determine the state of the satellite in its orbit at a specified epoch.[9] The initial conditions of one orbit can be adjusted at a given time to eliminate the secular divergence between the perturbed and unperturbed orbits to observe the periodic behavior.

In this analysis, two types of GPS satellite orbits were studied. The satellites of the Global Positioning System are distributed in six evenly spaced orbit planes. When completed, the final constellation will consist of 24 satellites at an orbit altitude of approximately 20,000 km with an orbit period of about 12 h. In this constellation, most satellites are exposed to full sunlight. As the orbit geometry changes, however, some GPS satellites will experience eclipsing or shadowing from the sun by the Earth. Both eclipsing and noneclipsing satellites are the focus of this study. Throughout its orbit, the GPS solar panel maintains a fixed orientation toward the sun. Nodal motion was not considered, because it is not significant for the short time interval of one week used in this study. No internally generated energy was modeled in this study, but the absorbed solar radiation that is converted to electricity was modeled, using the efficiency of the solar panel at 14.1%. Although studies have shown that for a TOPEX satellite, the thermal radiation forces originating with the spacecraft body are twice that due to the spacecraft solar panels, the major source for thermal reradiation forces on a GPS-like satellite are the spacecrafts's thin, large solar panels.[2,5] Consequently, in this analysis, the GPS satellite's main body was not considered. Other studies are currently considering this problem of modeling thermal reradiation forces for a complete GPS spacecraft.

Discussion of Results

To determine the direction and magnitude of the thermal force, the surface temperatures were calculated using the finite element method program, PDE-Protran.[7] Several simulations were tested. The data input that was required for the simulation is shown in Table 2. This table lists the material properties for a Block II GPS satellite solar panel.[8,10–14] The initial conditions included a solar panel orientation perpendicular to the sun and an initial temperature of 300 K. The time step used in the analysis was 100 s (one GPS orbit is approximately 43,200 s and the eclipsing period lasts approximately 3200 s).

GPS satellites experience an eclipsing season for only a few weeks every year. Eclipsing has a strong effect on the solar radiation environment of those satellites. This is evident in the temperature of a GPS satellite solar panel over one orbit shown in Fig. 3. The steady-state temperature for the sun-facing side is approximately 317 K and the shaded side is 313 K. These values compare well with the approximate value of 313 K that has been measured on the cold, shaded side of the solar panel for a GPS satellite.[10,11] The face exposed to the sun has not directly measured and therefore the temperature difference between surfaces is not well known, but is believed to be approximately 5 K.[10–12,15] During the eclipse period, which lasts approximately 1 h, there is a decline to panel temperature of approximately 253 K. Once exiting the shadow region, the solar panels slowly return to their steady-state temperatures after approximately 3 h.

Modeling the coverglass surface accurately has been difficult during this study because that information was not readily available. The thermal conductivity of this fused silica layer is very low as compared to that of two other dominant layers, the aluminum core and solar cell layers.[12] This layer, on the sun-facing side of the solar-panel, contributes most of the temperature imbalance primarily because of its low thermal conductivity and high thickness as compared to other solar panel layers, especially the aluminum core. Although it is believed that the solar panel coverglass layer is transparent to all incident radiation, the material properties from this specific layer of the solar panel were not removed from this analysis. It was important to simulate the solar panel as it exists in orbit to observe the long-term orbital effects of the thermal imbalance force on a GPS satellite. This is adequate as long as the correct material properties are used in the analysis.

As an example, two simulations were performed using identical solar panel parameters (values given in Table 2) except for different thermal conductivities for the sun-facing coverglass layer. These simulations are presented to show the sensitivity of the temperature and thermal force calculations to the thermal conductivity of the coverglass. The values for the thermal conducitivity given in test case 2 shown in Table 3 was used to demonstrate how unrealistic thermal forces can be computed when using incorrect values for the solar panel material properties. Previously, however, this was believed to be the correct value for the thermal conductivity of the fused silica coverglass layer of a GPS satellite solar panel.[8,13] The results shown in Table 3 describe the steady-state temperatures and thermal accelerations that were computed using the specified values for the coverglass thermal conductivity. Again, both test cases shown in Table 3 are identical except for the value of thermal conductivity for the solar panel coverglass layer.

In this paper, the reference frame is defined as spacecraft-centered radial and along-track components. The along-track component is also referred to as the transverse or down-track direction, defined in the direction of the satellite velocity vector. Figure 4 shows radial and along-track components of the acceleration due to thermal reradiation over one orbit for an eclipsing satellite. These com-

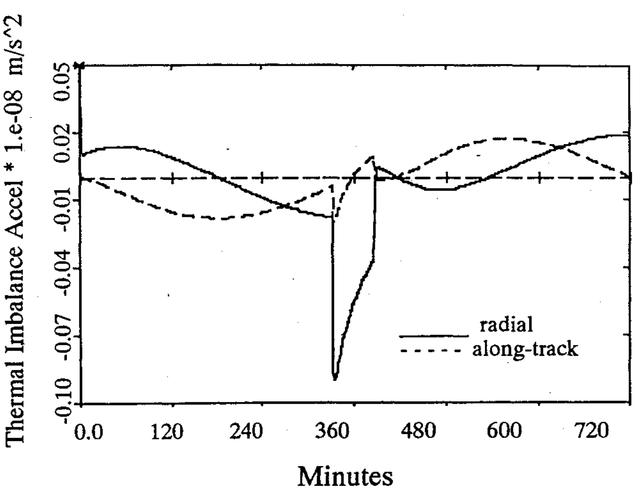

Fig. 4 Radial and along-track components for the thermal force over one orbit.

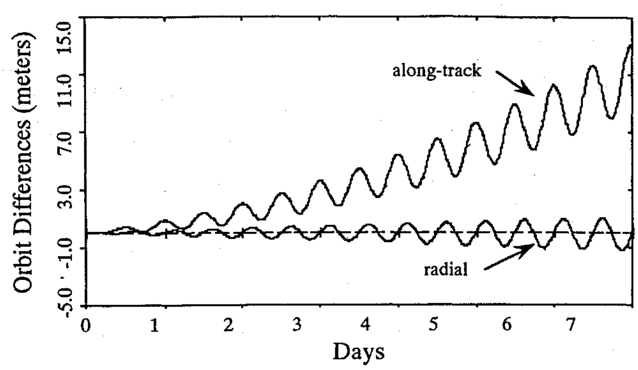

Fig. 5 Radial and along-track orbit differences, eclipsing satellite.

pare well with studies that have shown unmodeled nongravitational forces to cause errors of this magnitude.[15] Also, these results were computed using the information presented in Table 2 and described as test case 1 in Table 3.

Figure 5 shows the differences between two orbits, one computed using two-body effects only and another trajectory computed with two-body and thermal imbalance force for a satellite in an eclipsing orbit during one week. The radial rms is 0.5 m and the along-track rms is 5.2 m. These results were computed using a technique similar to the method used to predict satellite orbits based on a set of initial conditions and a complete force model of the spacecraft, which could include the solar radiation pressure and thermal imbalance force. In this case for an eclipsing satellite after seven days, the along-track components differ by approximately 13 m.

Figure 6 also shows the differences between two orbits, one with two-body effects only and another computed using two-body and thermal imbalance force for a satellite that is not in an eclipsing plane. The radial rms is 0.5 m and the along-track rms is 1.6 m. It can be seen from these results that an eclipsing satellite experiences a larger perturbation in the along-track direction over the span of one week than a satellite that is not in an eclipsing orbit plane. For the noneclipsing satellite case, after seven days, the along-track difference is approximately 5 m.

The next two figures represent the results computed using a least squares estimation algorithm in which the simulated observation data contained only the two-body gravitational and thermal imbalance reradiation forces. The force model used in the estimation algorithm contained the two-body gravitational force model with a solar radiation pressure model to observe the ability of the force model to account for thermal imbalance forces which have been difficult to model but exist in the observations. The best estimate of the satellite epoch state, in the least squares sense, is calculated using the satellite position, velocity, and a solar radiation pressure scale factor.

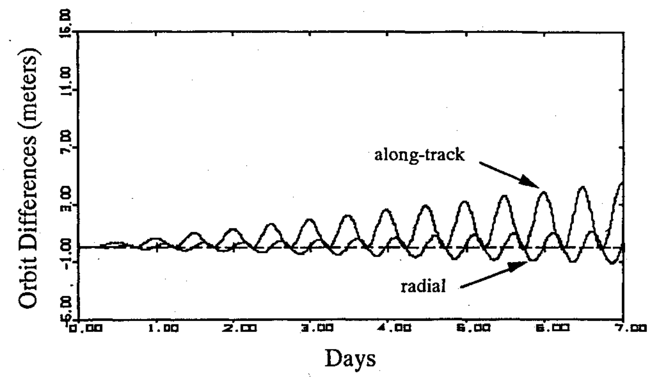

Fig. 6 Radial and along-track orbit differences, noneclipsing satellite.

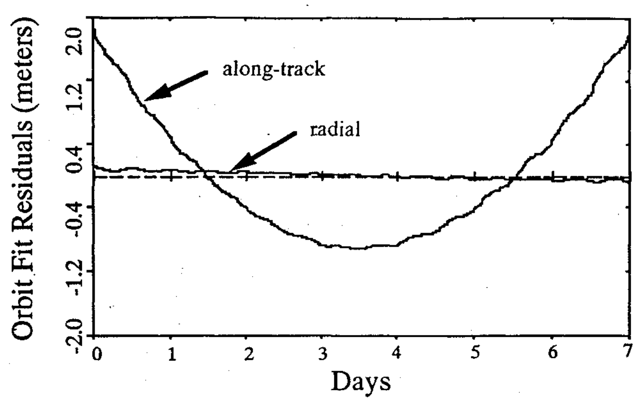

Fig. 7 Orbit fit residuals, with a solar radiation pressure scale factor, eclipsing satellite.

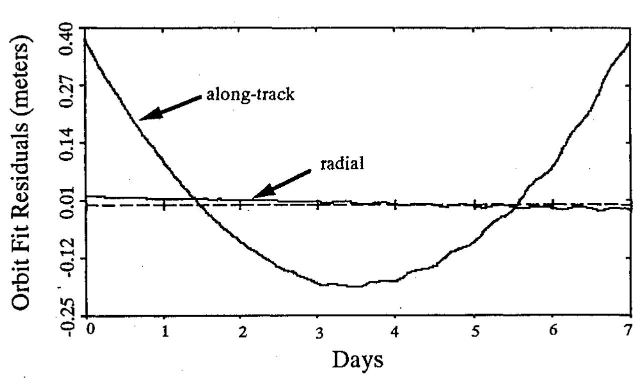

Fig. 8 Orbit fit residuals, with a solar radiation pressure scale factor, noneclipsing satellite.

Figure 7 shows the orbit fit residuals for a satellite in an orbit plane that is regularly eclipsing. The radial rms is 5 cm and the along-track rms is 80 cm. After seven days, the along-track orbit error is almost 2 m. These results show that the solar radiation pressure scale factor in the estimation scheme is capable of absorbing most of the orbit error due to thermal reradiation, but not all of the orbit error, especially in the along-track direction.

Figure 8 also shows orbit fit residuals for a GPS satellite, using the same estimation technique, but the satellite is in a noneclipsing orbit plane. The radial rms is 9 mm and the along-track rms is 17 cm. After seven days, the along-track orbit error is approximately 40 cm. Clearly, the eclipsing of the satellites has an influence on the orbit errors, when a thermal reradiation force is not included in the estimation force model. Larger orbit errors are calculated when the satellite is in an eclipsing orbit plane.

A one-week prediction can be made using the satellite state computed for the best least squares estimate in Fig. 7 and compared to the best least square estimate for that predicted week. Studies have shown that, for eclipsing satellites, the quadratic-like growth in the along-track direction can give errors as large as 50 m after a one-week prediction.[16]

Concluding Remarks

The current analysis has shown that orbit errors larger than 10 m occur when mismodeling nongravitational forces such as the thermal imbalance force presented here. A finite element method technique has been used to calculate satellite solar panel temperatures that are used to determine the magnitude and direction of the thermal imbalance force. Although this force may not be responsible for all of the force mismodeling, conditions may work in combination with the thermal imbalance force to produce such accelerations on the order of $1.0\,\mathrm{E}-9$ m/s^2. One possible contribution that is currently being studied is the solar panel misalignment, acting together with the thermal imbalance force, which may account for much of the unmodeled perturbations. If submeter accurate orbits and centimeter-level accuracy for geophysical applications are desired, a time-dependent model of the thermal imbalance force should be used, especially when satellites are eclipsing, where the observed errors are larger than for satellites in noneclipsing orbits. One study has shown that estimating additional stochastic solar radiation parameters improves GPS orbit accuracy significantly, especially for eclipsing satellites.[17] This technique can be used to absorb the orbit error that is caused by mismodeling thermal imbalance forces.

Although modeling the spacecraft solar panels alone may be considered insufficient, thermal force modeling of the entire spacecraft is a complicated problem. This has been done for spacecraft such as TOPEX where precise orbit determination is critical to mission success.[2] The study presented, here, however, focused only on modeling the solar panels where the material composition is not nearly as complex. Also, the problem of radiation absorbed and conducted through the solar panel and reradiated out is a simple one-dimensional time-dependent heat transfer problem, with no internal heat generation from scientific instruments or electronics.

Nongravitational perturbations like the thermal imbalance force have been observed for years on satellites like LAGEOS, and are still not completely understood. Thermal forces are dependent on the environment and specifically on such parameters as the satellite mass, cross-sectional area, and material composition. Unfortunately, these parameters can change or degrade with long-term exposure in space. For this reason, it may be more appropriate to estimate stochastic force parameters to represent the thermal reradiation forces since the nature and rate of material degradation of the satellite in orbit is unknown.[17] The results obtained using the finite element model used in this study agree with the work of others who have conducted similar studies using the finite difference technique to determine spacecraft thermal gradient forces in an effort to improve the satellite force models.

Acknowledgments

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